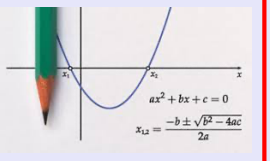
Math 125

Spring 2022

Lecture 23



Class QZ 16

1) Simplify: $(3-2i)(4+3i)$

$= 12 + 9i - 8i - 6i^2$

$= 12 + i - 6(-1) = \boxed{18+i}$ ✓

2) Divide: $\frac{5i}{1+2i} = \frac{5i(1-2i)}{(1+2i)(1-2i)}$

$= \frac{5i - 10i^2}{1 - 2i + 2i - 4i^2} = \frac{5i - 10(-1)}{1 - 4(-1)}$

$= \frac{5i + 10}{1+4} = \frac{10}{5} + \frac{5}{5}i$ ✓

$= \boxed{2+i}$

Simplify

Assume all variables are non-negative.

1) $\sqrt{20x^9y^8} = \sqrt[2]{4x^8y^8}\sqrt{5x}$

$= \boxed{2x^4y^4\sqrt{5x}}$

2) $\sqrt[3]{81x^{10}y^5} = \sqrt[3]{27x^9y^3}\sqrt[3]{3xy^2}$

$27 = 3^3$

$= \sqrt[3]{3^3x^9y^3}\sqrt[3]{3xy^2} = \boxed{3x^3y\sqrt[3]{3xy^2}}$

3) $3\sqrt{40x^3} - x\sqrt{1000x}$

$= 3\sqrt{4x^2}\sqrt{10x} - x\sqrt{100}\sqrt{10x}$

$= 3 \cdot 2x\sqrt{10x} - 10x\sqrt{10x}$

$= 6x\sqrt{10x} - 10x\sqrt{10x} = \boxed{-4x\sqrt{10x}}$

Simplify

1) $3\sqrt{6}(2\sqrt{3} - 3\sqrt{2})$

$= 6\sqrt{18} - 9\sqrt{12} = 6\sqrt{9}\sqrt{2} - 9\sqrt{4}\sqrt{3}$

$= 6 \cdot 3\sqrt{2} - 9 \cdot 2\sqrt{3}$

$= \boxed{18\sqrt{2} - 18\sqrt{3}}$

2) $(3\sqrt{5} - \sqrt{2})^2$

$= (3\sqrt{5} - \sqrt{2})(3\sqrt{5} - \sqrt{2})$

$= 9\sqrt{25} - 3\sqrt{10} - 3\sqrt{10} + \sqrt{4} = 9 \cdot 5 - 6\sqrt{10} + 2$

$= \boxed{47 - 6\sqrt{10}}$

3) $(\sqrt[3]{5} - \sqrt[3]{2})(\sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4})$

$= \sqrt[3]{125} + \cancel{\sqrt[3]{50}} + \cancel{\sqrt[3]{20}} - \cancel{\sqrt[3]{50}} - \cancel{\sqrt[3]{20}} - \sqrt[3]{8}$

$= 5 - 2 = \boxed{3}$

Solve & check

$x - \sqrt{x+6} = 0$

$x = \sqrt{x+6}$

$(x)^2 = (\sqrt{x+6})^2$

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0$ OR $x + 2 = 0$

$x = 3$ ~~$x = -2$~~ E.S.

$\{3\}$

Check:

$3 - \sqrt{3+6} = 0$

$3 - \sqrt{9} = 0$

$3 - 3 = 0$

$0 = 0$ ✓

$-2 - \sqrt{-2+6} = 0$

$-2 - \sqrt{4} = 0$

$-2 - 2 = 0$

$-4 = 0$

False

Rationalize the denominator:

1) $\frac{2}{\sqrt{5}} = \frac{2 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \boxed{\frac{2\sqrt{5}}{5}}$

2) $\frac{2}{\sqrt[3]{5}} = \frac{2 \cdot \sqrt[3]{5^2}}{\sqrt[3]{5} \cdot \sqrt[3]{5^2}} = \frac{2\sqrt[3]{25}}{\sqrt[3]{5^3}} = \boxed{\frac{2\sqrt[3]{25}}{5}}$

3) $\frac{xy}{\sqrt[5]{x^2y^4}} = \frac{xy \cdot \sqrt[5]{x^3y^1}}{\sqrt[5]{x^2y^4} \cdot \sqrt[5]{x^3y^1}} = \frac{xy\sqrt[5]{x^3y}}{\sqrt[5]{x^5y^5}}$

$= \frac{xy\sqrt[5]{x^3y}}{xy}$

$= \boxed{\sqrt[5]{x^3y}}$

Rationalize the deno:

1) $\dfrac{\sqrt{6}}{\sqrt{2}+1} = \dfrac{\sqrt{6}(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \dfrac{\sqrt{12}-\sqrt{6}}{\sqrt{4}-\cancel{\sqrt{2}}+\cancel{\sqrt{2}}-1}$

$= \dfrac{\sqrt{4}\sqrt{3}-\sqrt{6}}{1} = \boxed{2\sqrt{3}-\sqrt{6}}$ $\quad 2-1=1$

2) $\dfrac{2\sqrt{5}}{\sqrt{10}-\sqrt{3}}$

$= \dfrac{2\sqrt{5}(\sqrt{10}+\sqrt{3})}{(\sqrt{10}-\sqrt{3})(\sqrt{10}+\sqrt{3})} = \dfrac{2\sqrt{50}+2\sqrt{15}}{\sqrt{100}+\cancel{\sqrt{30}}-\cancel{\sqrt{30}}-\sqrt{9}}$

$= \dfrac{2\sqrt{25}\sqrt{2}+2\sqrt{15}}{10-3}$

$= \boxed{\dfrac{10\sqrt{2}+2\sqrt{15}}{7}}$

3) $\dfrac{\sqrt{6}-\sqrt{3}}{\sqrt{6}+\sqrt{3}} = \dfrac{(\sqrt{6}-\sqrt{3})(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})}$

$= \dfrac{\sqrt{36}-\sqrt{18}-\sqrt{18}+\sqrt{9}}{\sqrt{36}-\cancel{\sqrt{18}}+\cancel{\sqrt{18}}-\sqrt{9}}$

$= \dfrac{6-2\sqrt{18}+3}{6-3} = \dfrac{9-2\sqrt{9}\sqrt{2}}{3}$

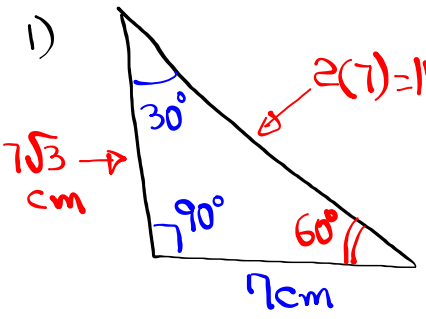
$= \dfrac{9-6\sqrt{2}}{3}$

$= \dfrac{\cancel{3}(3-2\sqrt{2})}{\cancel{3}}$

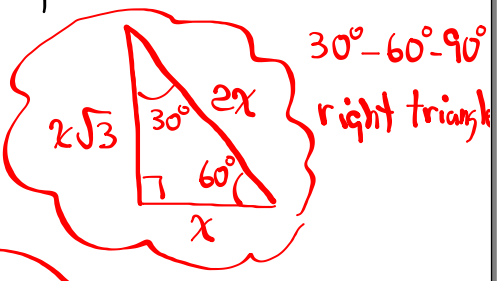
$= \boxed{3-2\sqrt{2}}$

Solve the following triangles:

1)



$2(7) = 14$ cm



30°-60°-90° right triangle

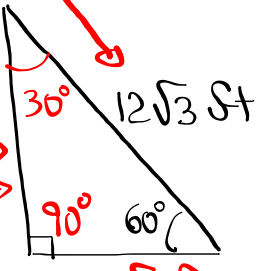
$2x = 12\sqrt{3}$

$x = 6\sqrt{3}$

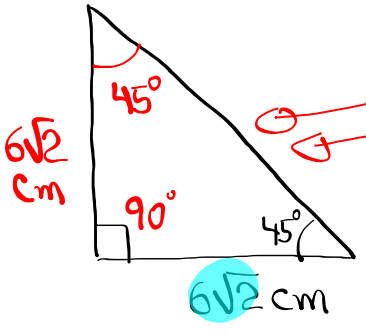
$2\sqrt{3} = 6\sqrt{3}\sqrt{3}$

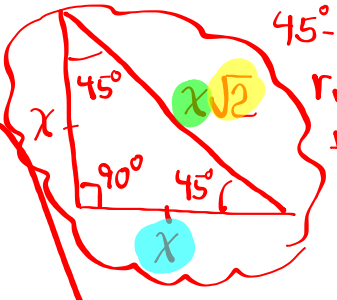
$= 6\sqrt{9} = 6 \cdot 3 = 18$

2)



3)





45°-45°-90° right triangle

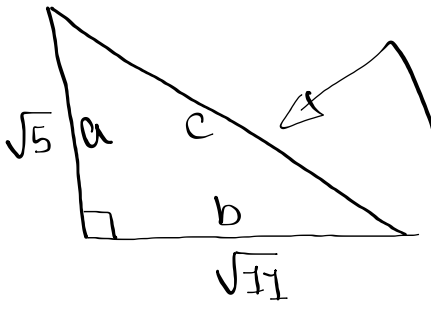
$6\sqrt{2}$ $\sqrt{2}$

$= 6\sqrt{4}$

$= 6 \cdot 2$

$= 12$ cm

Find the hypotenuse: use pythagorean thrm



$a^2 + b^2 = c^2$

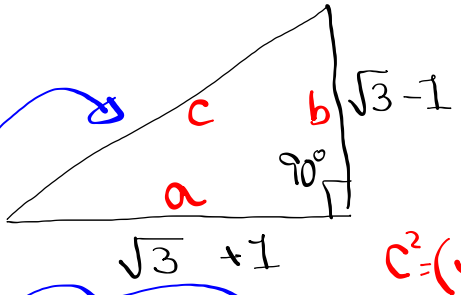
$(\sqrt{5})^2 + (\sqrt{11})^2 = c^2$

$5 + 11 = c^2$

$c^2 = 16$

$c = 4$

Find the hypotenuse:



Right Triangle

$c^2 = a^2 + b^2$

$c^2 = (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2$

$c^2 = (\sqrt{3}+1)(\sqrt{3}+1) + (\sqrt{3}-1)(\sqrt{3}-1)$

$c^2 = \sqrt{9} + \sqrt{3} + \sqrt{3} + 1 + \sqrt{9} - \sqrt{3} - \sqrt{3} + 1$

$= 3 + 1 + 3 + 1$

$c^2 = 8$

$c = \sqrt{8} = \sqrt{4}\sqrt{2}$

$c = 2\sqrt{2}$

Be aware of due dates of SG online.

If $x^2 = K$, then $x = \pm\sqrt{K}$.

Square-Root Method

1) Solve $x^2 = 36$

By S.R.M., $x = \pm\sqrt{36}$ $\boxed{x = \pm 6}$

$\{\pm 6\}$

2) Solve $x^2 - 4 = 36$

$x^2 = 36 + 4$

$x^2 = 40$

Use S.R.M. $x = \pm\sqrt{40}$

$x = \pm\sqrt{4}\sqrt{10}$ $\{\pm 2\sqrt{10}\}$

$\boxed{x = \pm 2\sqrt{10}}$

3) Solve $x^2 + 100 = 0$

$x^2 = -100$

By S.R.M. $x = \pm\sqrt{-100}$

$= \pm\sqrt{100}\sqrt{-1}$

$\boxed{x = \pm 10i}$ $\{\pm 10i\}$

4) Solve $x^2 + 100 = 25$

$x^2 = 25 - 100$

$x^2 = -75$

By S.R.M.

$x = \pm\sqrt{-75}$

$x = \pm\sqrt{25}\sqrt{3}\sqrt{-1}$

$\boxed{x = \pm 5\sqrt{3}i}$

$\{\pm 5\sqrt{3}i\}$

5) Solve $(x-2)^2 = 81$

By S.R.M.

$x-2 = \pm\sqrt{81}$

$x = 2 \pm 9$

$x = 2+9 = 11$

$x = 2-9 = -7$

$\{-7, 11\}$

6) Solve $(x+2)^2 = -81$

By S.R.M.

$x+2 = \pm\sqrt{-81}$

$x+2 = \pm\sqrt{81}\sqrt{-1}$

$x+2 = \pm 9i$

$x = -2 \pm 9i$

$\{-2 \pm 9i\}$

7) Solve $(5x-3)^2 = -16$

By S.R.M.

$5x-3 = \pm\sqrt{-16}$

$5x-3 = \pm 4i$

$5x = 3 \pm 4i$

$x = \frac{3}{5} \pm \frac{4}{5}i$

$\left\{\frac{3}{5} \pm \frac{4}{5}i\right\}$

Quadratic Equation: $ax^2+bx+c=0$, $a\neq 0$

b^2-4ac is called discriminant.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ Quadratic Formula

① $2x^2-3x-5=0$

$a=2$ $b=-3$ $c=-5$

$b^2-4ac=(-3)^2-4(2)(-5)=9-(-40)=49$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-3)\pm\sqrt{49}}{2(2)}=\frac{3\pm 7}{4}$$

$x=\frac{3+7}{4}=\frac{10}{4}=\boxed{\frac{5}{2}}$ $x=\frac{3-7}{4}=\frac{-4}{4}=\boxed{-1}$ $\left\{-1,\frac{5}{2}\right\}$

② $x^2+4x+5=0$

$ax^2+bx+c=0$

$a=1$ $b=4$ $c=5$

$b^2-4ac=4^2-4(1)(5)=16-20=-4$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-4\pm\sqrt{-4}}{2(1)}=\frac{-4\pm 2i}{2}$$

$$=\frac{-4}{2}\pm\frac{2}{2}i=\boxed{-2\pm i}$$ $\{-2\pm i\}$

3) $x^2 = 8 - 10x$

$ax^2 + bx + c = 0$

$x^2 - 8 + 10x = 0$

$x^2 + 10x - 8 = 0$

$a=1 \quad b=10 \quad c=-8$

$b^2 - 4ac = 10^2 - 4(1)(-8) = 100 + 32 = 132$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-10 \pm \sqrt{132}}{2(1)} = \frac{-10 \pm \sqrt{4}\sqrt{33}}{2}$

$= \frac{-10 \pm 2\sqrt{33}}{2} = \frac{-10}{2} \pm \frac{2\sqrt{33}}{2} = \boxed{-5 \pm \sqrt{33}}$

$\{-5 \pm \sqrt{33}\}$

4) Solve $(3x+2)(2x+1) = 15$

Foil, Simplify, write in $ax^2+bx+c=0$ form

$6x^2 + 3x + 4x + 2 - 15 = 0$

$6x^2 + 7x - 13 = 0$

$a=6, \ b=7, \ c=-13$

$b^2 - 4ac = 7^2 - 4(6)(-13) = 361$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-7 \pm \sqrt{361}}{2(6)} = \frac{-7 \pm 19}{12}$

$x = \frac{-7+19}{12} = \frac{12}{12} = \boxed{1} \qquad x = \frac{-7-19}{12} = \frac{-26}{12} = \boxed{\frac{-13}{6}}$

$\{\frac{-13}{6}, 1\}$

Find two consecutive integers such that the sum of their squares is 61.

x, $x+1$

$x^2 + (x+1)^2 = 61$

$x^2 + x^2 + 2x + 1 = 61$

$2x^2 + 2x + 1 - 61 = 0$

$2x^2 + 2x - 60 = 0$

Divide by 2

$x^2 + x - 30 = 0$

$ax^2 + bx + c = 0$

$a=1 \quad b=1 \quad c=-30$

$b^2 - 4ac = 1^2 - 4(1)(-30)$

$= 121$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{121}}{2(1)} = \frac{-1 \pm 11}{2}$

$x = \frac{-1+11}{2} = \frac{10}{2} = 5$

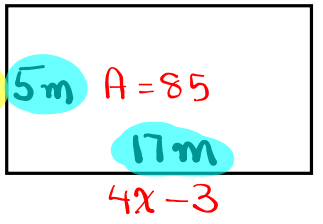
$x = \frac{-1-11}{2} = \frac{-12}{2} = -6$

x	$x+1$
5	6
-6	-5

Length of a rectangle is 3 m shorter than 4 times its width.

Its area is 85 m^2.

Find its dimensions.



$A = 85$

$LW = 85$

$x(4x-3) = 85$

$4x^2 - 3x - 85 = 0$

$a=4 \quad b=-3 \quad c=-85$

$b^2 - 4ac = (-3)^2 - 4(4)(-85)$

$= 1369$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-3) \pm \sqrt{1369}}{2(4)} = \frac{3 \pm 37}{8}$

$x = \frac{3+37}{8} = \frac{40}{8} = 5$

$x = \frac{3-37}{8} = \frac{-34}{8}$

Class QZ 17

Rationalize the denominator

1) $\frac{-2}{\sqrt{6}} = \frac{-2\sqrt{6}}{\sqrt{6}\sqrt{6}}$

$= \frac{-2\sqrt{6}}{\sqrt{36}}$

$= \frac{-2\sqrt{6}}{6}$

$= \boxed{\frac{-\sqrt{6}}{3}}$

2) $\frac{3}{\sqrt{6}+\sqrt{3}} = \frac{3(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})}$

$= \frac{3(\sqrt{6}-\sqrt{3})}{\sqrt{36}-\sqrt{18}+\sqrt{18}-\sqrt{9}}$

$= \frac{3(\sqrt{6}-\sqrt{3})}{6-3}$

$= \boxed{\sqrt{6}-\sqrt{3}}$